Supersymmetric Unification: a mini-review of recent developments*

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Abstract

Some recent results in supersymmetric grand unified theories are reviewed.

1. Introduction

The Standard Model (SM) of particle interactions has proven very successful in describing collider physics. To go beyond, one is interested in discovering new states or eliminating free parameters of the SM. Usually each of these avenues involves new or larger symmetries. The idea of supersymmetric (SUSY) grand unified theories (GUTs) is no exception: the gauge group of the Standard Model is included in a larger grand unified group and a new symmetry relating particles of different spin is introduced. This approach then predicts the existence of many new states (the sparticles) and can eliminate free parameters (e.g. $\sin\theta_W$) that exist in the Standard Model.

Not long after the unification of the electroweak interactions the idea of a further unification of all the forces in the Standard Model into a single gauge group was born[1]. Shortly thereafter the implications for the value of the weak mixing angle was derived[2]. The low-energy theory was then made supersymmetric[3] yielding a slightly different value for $\sin\theta_W$. Before the precision data from LEP both these versions of a grand unified theory incorporating a desert between the electroweak and the GUT scale were experimentally viable. With the recent precise electroweak data, the supersymmetric version alone can describe the experimental results in the absence of an intermediate scale[4].

The other development that is central to the success of SUSY unification is electroweak symmetry breaking. Supersymmetry protects the hierarchy of the electroweak scale and the GUT scale; in addition the breakdown of the electroweak symmetry can be understood when the supersymmetry is local[5]. The large value of the top quark Yukawa coupling enhances large logs that cause the spontaneous breakdown of the electroweak symmetry[6].

Following the revival of interest in SUSY GUTs caused by the unification of the gauge couplings, some of the more model-dependent earlier predictions were reinvestigated. In particular, the relation between the bottom quark and the tau lepton mass that occurs in some minimal versions of grand unified theories was updated. The equality of b and τ Yukawa couplings at the GUT scale was first proposed in Ref. [7]. The importance of a large top Yukawa coupling in suppressing the bottom quark mass was emphasized by Ibañez and Lopez[8]. More recently the correlation between the bottom quark mass and the tau lepton mass was explored, and an inconsistency in the nonsupersymmetric desert prediction was uncovered[9]. Moreover b-τ coupling unification is perfectly viable in the minimal supersymmetric extension to the Standard Model. While Yukawa coupling unification is not as general as that of gauge coupling unification (it involves specific assumptions about the GUT symmetry breakdown), the success of the simplest relation added extra impetus to the interest in supersymmetric GUTs. The m_b/m_τ relation implies that the top quark Yukawa coupling is probably at its infrared quasi-fixed point, which in turn limits the relation of the top quark and the

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ratio of the Higgs vevs in supersymmetry to a narrow corridor of values[10]–[13]. When the uncertainty on the top quark mass from CDF is reduced, then only small (≈ 1) and large ($\approx m_t/m_b$) values of $\tan\beta$ will be allowed, with the small $\tan\beta$ solution preferred. [The situation is complicated somewhat by the presence of threshold corrections at the electroweak and grand unified scales—if large enough and if $\alpha_3(M_Z)$ is at the lower end of its experimentally allowed range, then the fixed-point solution might not apply.] The relationship implied by the top quark Yukawa coupling fixed point on the value of $\tan\beta$ also occurs in a top quark condensate mode[14] when the scale Λ is near the GUT scale.

2. Evolution of Dimensionless Couplings

The gauge couplings evolve according to renormalization group equations (RGE) with the solution

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(M_G) - \frac{b_i}{2\pi} t \; . \tag{1}$$

at the one-loop level where $t = \ln(Q/M_G)$ defines the scale. The parameters b_i are determined by the particle content of the effective theory.

The evolution of the top quark Yukawa coupling is described by the RGE,

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{16\pi^2}\left[-\frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + 6\lambda_t^2 + \lambda_b^2\right] \; . \tag{2}$$

Figures 1 and 2 show the solution of these renormalization group equations for values of the bottom quark running mass. One sees that the top Yukawa coupling is driven to its infrared fixed point[10]–[21].

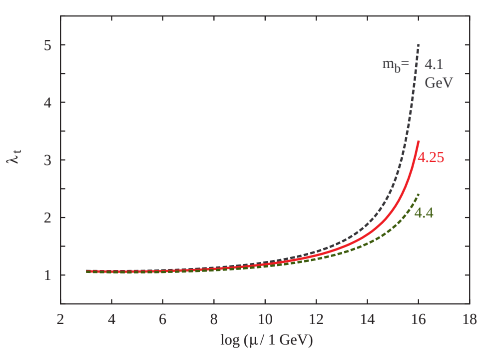

Fig. 1. If λ_t is large at M_G, then the renormalization group equation causes $\lambda_t(Q)$ to evolve rapidly towards an infrared fixed point as $Q \to m_t$ (from Ref. [10]).

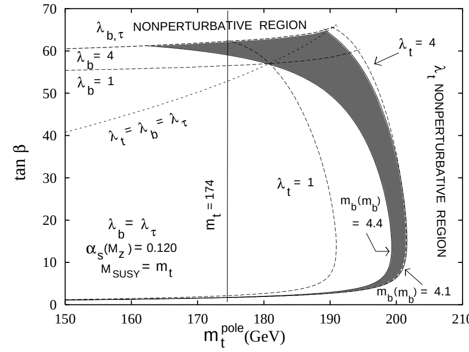

Fig. 2. Contours of constant $m_b(m_b)$ in the $m_t(m_t), \tan\beta$ plane with contours of constant GUT scale Yukawa couplings (adapted from Ref. [10]).

Converting this relation for the top quark pole mass yields[19]

$$m_t^{\rm pole} \approx (200{\rm GeV}) \sin\beta \; . \tag{3}$$

The value of 200 GeV that appears in the above equation is subject to some uncertainty due to varying $\alpha_3(M_Z)$ and to threshold effects[13, 22].

Adding a singlet N to the minimal supersymmetric model provides an additional coupling $\lambda H_1 H_2 N$ in the superpotential, which could conceivably suppress the bottom quark mass sufficiently that the top quark Yukawa coupling could be reduced. However constraints on the perturbativity of the coupling λ preserves the fixed point condition[20, 21]. A four generation model is not easily compatible with Yukawa unification[23].

3. RGE Evolution of Sparticle Masses

An attractive property of models based on supergravity is that the symmetry breakdown in the electroweak sector can be attributed to large logs that contribute to the Higgs potential[24]–[34]. One must arrive at the correct scale for the electroweak interactions without breaking color or charge. This is accomplished by imposing two minimization conditions obtained from the Higgs potential.

The minimum of the Higgs potential must occur by the acquisition of vacuum expectation values. Minimizing the tree-level potential with respect to the two neutral CP-even Higgs degrees of freedom yields the two conditions

$$\frac{1}{2}M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \; . \tag{4}$$

$$-B\mu = \frac{1}{2}(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2)\sin 2\beta \; . \tag{5}$$

where m_{H_1} and m_{H_2} are soft-supersymmetry breaking parameters and μ is the Higgs mass in the superpotential. In order to avoid large cancellations between the

terms on the right-hand side of Eq. (4), some naturalness criteria are inposed, which in turn typically imply that the sparticle masses are not too high ($\lesssim 1$ TeV).

For the low $\tan\beta$ fixed-point solution as few as two inputs for the soft-supersymmetry breaking parameters are needed—a universal scalar mass m_0 and a common gaugino mass $m_{1/2}$[30]. The low energy sparticle mass can be given in terms of these inputs.

The heaviest chargino and the two heaviest neutralino states are primarily Higgsino with masses approximately equal to $|\mu|$. Typical mass relationships are displayed in Figure 3.

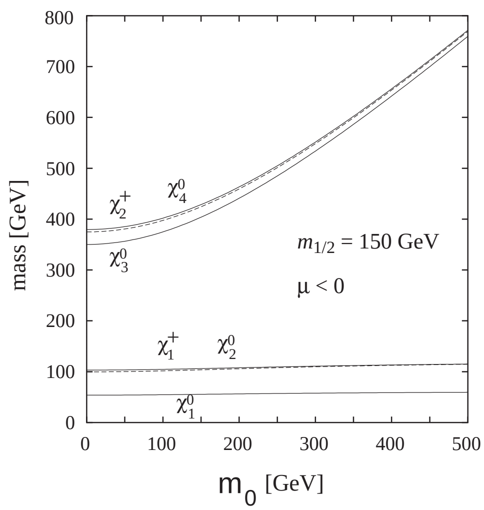

Fig. 3. The chargino and neutralino masses are plotted versus m_0 for $m_{1/2} = 150$ GeV for a low value of $\tan\beta$ and $\mu < 0$ (from Ref.[31]).

For the low-$\tan\beta$ solution, the mass of the lightest Higgs h comes mainly from radiative corrections[19, 21, 35, 36, 37, 38]. Experiments at LEP II will cover the region $m_h \lesssim \sqrt{s}/2$. Recent work[39] has shown that a $h \to b\bar{b}$ search at the Tevatron may be possible. The heavy Higgs states are (approximately) degenerate $\approx M_A$; see Figure 4. The Higgs discovery potential at e^+e^- colliders has recently been discussed[40].

The squark and slepton masses also display simple asymptotic behavior at large $|\mu|$; see Figure 5. The first and second squark generations are approximately degenerate. The splitting of the stop quark masses grows as $|\mu|$ increases and the lightest stop can be as light as 45 GeV (or even lighter with fine tuning). The masses could be much larger than is indicated in the figure since the value of m_0 could be large.

While the universality of the scalar masses has been assumed for the Figures presented above, recently there has been much interest in considering the implications of nonuniversality on the supersymmetric spectrum and on reconsidering the constraints from flavor changing neutral currents[41].

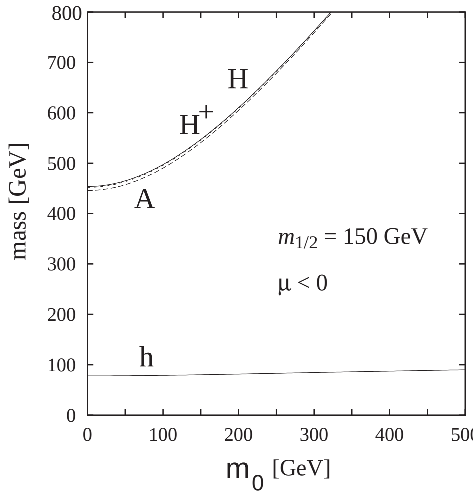

Fig. 4. The supersymmetric Higgs masses are plotted versus m_0 for $m_{1/2} = 150$ GeV for a low value of $\tan\beta$ and $\mu < 0$ (from Ref.[31]).

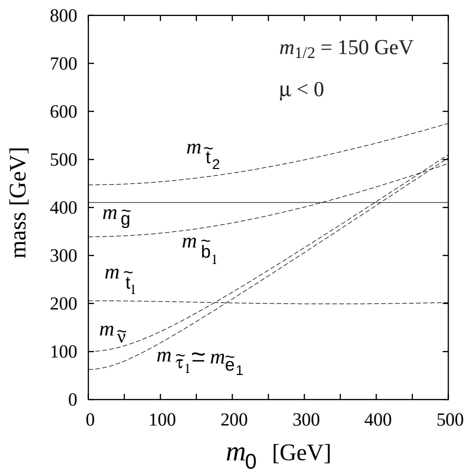

Fig. 5. The squark and slepton masses are plotted versus m_0 for $m_{1/2} = 150$ GeV for a low value of $\tan\beta$ and $\mu < 0$.

The MSSM has an R-parity symmetry so the lightest supersymmetric particle (LSP) is stable. Usually the LSP is the lightest neutralino, but for small values of m_0 the supersymmetric partner of the tau lepton can be lighter. For the lightest particle to be neutral, as required, there is an upper bound on the value of $m_{1/2}$ for small m_0. In particular such an upper bound exists for no-scale models ($m_0 = 0$), and is more stringent for $\mu > 0$ due to the mixing between the left- and right-handed $\tilde{\tau}$. The phenomenological issues of the Yukawa unified no-scale model have been examined in Ref. [42].

The LSP can naturally account for the dark matter of the Universe[43, 44]. Large values of μ result in the lightest neutralino being predominantly gaugino. This leads to a reduced rate of annihilation of neutralinos and can provide too much relic abundance and overclose the Universe. However the s-channel h pole can enhance the annihilation rate and rescue the dark matter explanation[45].

4. Proton Decay

One of the major additions to particle physics from the concept of grand unified theories is that the proton might be unstable. Stringent experimental limits on proton decay rule out many models, including the nonsupersymmetric version of SU(5). At first sight it might appear that the supersymmetric versions of SU(5) are safe, since the GUT scale is considerably higher and therefore proton decay occurring through the dimension six operators is much suppressed. However, dangerous dimension five operators are introduced in the supersymmetric versions of grand unified models. In the minimal supersymmetric SU(5) GUT, one must have very heavy sleptons to avoid the proton decay bound[46, 47]. However, other models can greatly suppress or eliminate entirely proton decay.

5. Possibilities for Experimental Searches

There are many interesting signals for supersymmetry at present and future colliders. The missing p_T signal at the Tevatron or at the LHC is a classic experimental signature of supersymmetry. If the charginos and the neutralinos are sufficiently light, then trilepton signals are possible[48]:

$$W^{\pm *} \to \chi_1^{\pm}\chi_2^0 \to \ell^{\pm}\ell^{+}\ell^{-}\chi_1^0\chi_1^0 \,. \tag{6}$$

Gluinos will be produced abundantly at hadron colliders, and decays can produce like-sign dilepton signals[49]. On the other hand, for some regions of parameter space the gluino may decay predominantly into stop [50]:

$$\tilde{g} \to \tilde{t}t \,. \tag{7}$$

If the stop is lighter than the top then stoponium bound states can be formed which subsequently decay into photon pairs or Higgs bosons[51]. A future high energy e^+e^- collider (NLC) would provide an opportunity to produce and study the properties of sleptons, charginos, and supersymmetric Higgs bosons[52].

6. Implications for $b \to s\gamma$ decay

The measured rate for the inclusive decay $b \to s\gamma$ [53]

$$\mathrm{BR}(B \to s\gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4} \tag{8}$$

is close to the SM prediction. The predicted rate in SUSY models for small $\tan\beta$ is somewhat larger than the SM for $\mu > 0$ and generally smaller than the SM rate for $\mu < 0$[54]. Figure 6 shows the general trend of contours for the inclusive rate for $\mu < 0$. Unfortunately the current theoretical uncertainty is at least $\pm 25\%$[56], so until further theoretical progress is made, one cannot determine the sign of μ. SUSY contributions to $B^0 - \overline{B}^0$, $D^0 - \overline{D}^0$ and $K^0 - \overline{K}^0$ could also be relevant to placing restrictions on models[57]. The implications of the $Z \to b\overline{b}$ measurements at LEP on supersymmetric unification have recently been investigated[58].

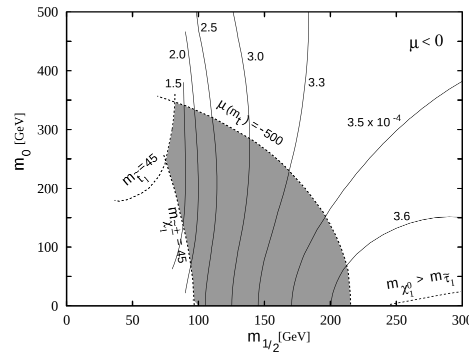

Fig. 6. Contour lines for the $b \to s\gamma$ inclusive rate for $\mu < 0$ (from Ref. [55]).

7. Conclusions

According to all RGE sparticle mass spectrum analyses, two broad conclusions about the implications of SUSY unification can be drawn:

- Interesting regions of the SUSY parameter space can be covered at the Tevatron with the main injector, and possibly further improvements in the luminosity or upgrades of the center-of-mass energy[59].
- The LHC and the NLC are guaranteed to be SUSY factories if supersymmetry exists; the task of determining how to pull the signals out of the backgrounds is continuing[60].

8. Acknowledgements

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